647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2021 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Question-1(A) Answer any two out of three. **[10]**

(1) X is metric space and $E \subset X$. Prove that $\overline{E}$ is closed.

(2) X is metric space. Show that union of finite numbers of closed subsets of X is closed. Explain with example that this statement is not true for infinite numbers of closed subsets of X.

(3) Explain cantor set and define it. Show that $\frac{1}{4} \in C$.

(B) Answer any one out of two. **[4]**

(1) (i) in usual notation show that $(A \cap B)^0 = A^0 \cap B^0$.

(ii) Explain with example that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$

(2) (i) R is discrete metric space. Find N(3,0.5) in R.

(ii) is set Q dense in R ? Explain.

(iii) Check whether set $E=(-\infty,5] \cup \{10\}$ is open and closed.

Question-2(A) Answer any two out of three. **[10]**

(1) Prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall$ $\varepsilon>0$ there exist partition P of [a,b] such that $U(P,f) - L(P,f) < \varepsilon$, Where $\|P\| < \delta$.

(2) If functions f and g are R-integrable in [a,b] then show that f+g is also R-integrable in [a,b] and

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

(3) Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$ Then show that $U(P^*,f) \leq U(P,f)$.

(B) Answer any one out of two. **[4]**

(1) Find $\int_0^1 x^2\, dx$ using definition of R-integration.

(2) If bounded function f on [a,b] is continuous then show that f is R-integrable.

Question-3(A) Answer any two out of three. **[10]**

(1) Using darbour's theorem find value of

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots\dots\dots\dots + \left(\frac{n}{n}\right)^2\right]$$

(2) State and prove general form of first mean value theorem for R-integration.

(3) G=R-{1}. Binary operation * is defined on G as ; for $a,b \in G$, a*b=a+b-ab then show that (G,*) is group and find 3^{-1} in G.

(B) Answer any one out of two. **[4]**

(1) Show that $\frac{\pi^2}{6} \leq \int_0^{\pi} \frac{x}{2+\cos(x)}\, dx \leq \frac{\pi^2}{2}$

(2) Define: Group , Binary operation. Show that equation a*x=b has unique solution in Group (G,*). Where $a,b \in G$.

Question-4(A) Answer any two out of three. **[10]**

(1) State and prove lagrange's theorem.

(2) Define: Symmetric group. Find number of elements of Symmetric group S_n. Also write all elements of S_3.

(3) Prove that group and its subgroup has same identity element. Also prove that inverse of any element in group and its subgroup is also same.

(B) Answer any one out of two. **[4]**

(1) Prove that H={x$\in$G/xa=ax} is subgroup of group G. where a is invariant element of G.

(2) $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ then find σ^{-1} and o(σ). Also check whether σ is even or odd.

Question-5(A) Answer any two out of three. **[10]**

(1) if $(G,*) \cong (G',*')$. Show that G is commutative iff G' is commutative. Where G and G' are groups.

(2) G is finite group and a ϵ G. prove that O(a)/O(G).

(3) Prove that every permutation f of finite set is a product of disjoint cycles.

(B) Answer any one out of two. **[4]**

(1) For n≥2, in usual notation show that A_n is normal subgroup of S_n.

(2) Prepare group table in usual notation for quotient group S_3/A_3.

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